

Topic: Sketching sine and cosine

Question: Where is the function zero, and where is it negative?

Consider the function $-4\cos(5\theta)$. At what angle(s) θ in the interval $[0, 2\pi/5)$ is the value of this function equal to 0, at what angle(s) θ is its value equal to 4, and where in the interval $[0, 2\pi/5)$ is this function negative?

Answer choices:

- A In the interval $[0, 2\pi/5)$, the value of the function $-4\cos(5\theta)$ is equal to 0 at $\theta = 0$ and $\theta = \pi/5$; its value is equal to 4 at $\theta = 3\pi/10$; and its value is negative in the interval $(0, \pi/5)$.
- B In the interval $[0, 2\pi/5)$, the value of the function $-4\cos(5\theta)$ is equal to 0 at $\theta = \pi/10$ and $\theta = 3\pi/10$; its value is equal to 4 at $\theta = \pi/5$; and its value is negative in the intervals $[0, \pi/10)$ and $(3\pi/10, 2\pi/5)$.
- C In the interval $[0, 2\pi/5)$, the value of the function $-4\cos(5\theta)$ is equal to 0 at $\theta = \pi/10$ and $\theta = 3\pi/10$; its value is equal to 4 at $\theta = 0$; and its value is negative in the interval $(\pi/10, 3\pi/10)$.
- D In the interval $[0, 2\pi/5)$, the value of the function $-4\cos(5\theta)$ is equal to 0 at $\theta = 0$ and $\theta = \pi/5$; its value is equal to 4 at $\theta = \pi/10$; and its value is negative in the interval $(\pi/5, 2\pi/5)$.



Solution: B

The function $-4 \cos(5\theta)$ is in the form $a \cos(b\theta)$ where $a = -4$ and $b = 5$. Thus the amplitude of this function is $|a|$ (i.e., 4), and the period is

$$\frac{2\pi}{b} = \frac{2\pi}{5}$$

Note that exactly one period of this function fits into the interval $[0, 2\pi/5)$. Since this is a cosine function, its value is equal to 0 at the angles θ that are one-fourth and three-fourths of the way through that interval, that is, at

$$\theta = \left(\frac{1}{4}\right) \left(\frac{2\pi}{5}\right) = \frac{\pi}{10}$$

and

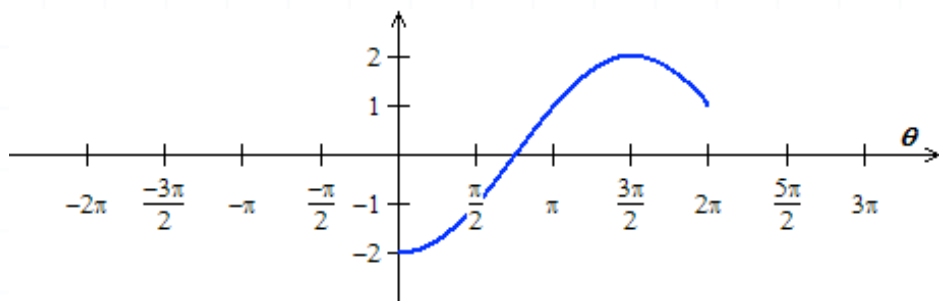
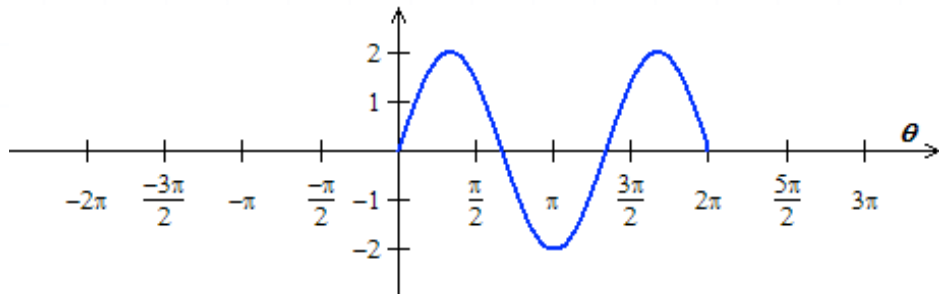
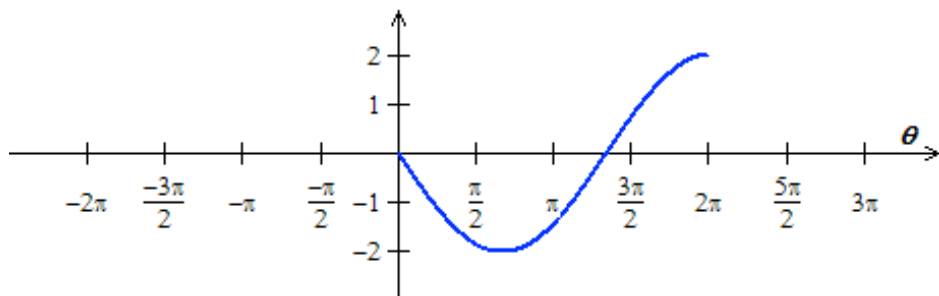
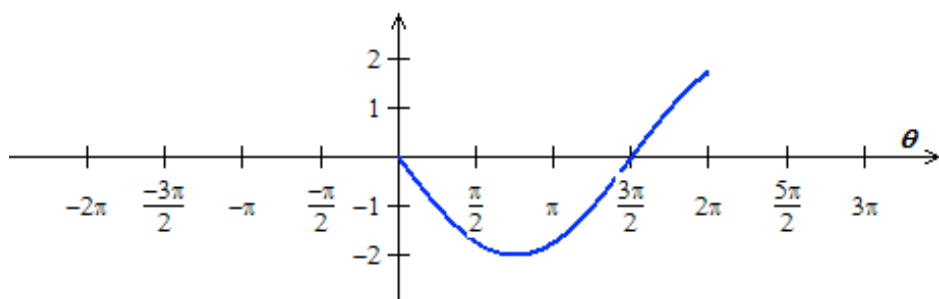
$$\theta = \left(\frac{3}{4}\right) \left(\frac{2\pi}{5}\right) = \frac{3\pi}{10}$$

Since a is negative, the value of this function is equal to 4 (i.e., $-a$) at the angle θ which is halfway through the interval $[0, 2\pi/5)$, that is, at $\theta = \pi/5$. Also, the function is negative in the intervals $[0, \pi/10)$ and $(3\pi/10, 2\pi/5)$.



Topic: Sketching sine and cosine**Question:** Identify the graph of the function.

$$-2 \sin(2\theta/3) \text{ over the interval } [0, 2\pi)$$

Answer choices:**A****B****C****D**

Solution: D

The function $-2 \sin(2\theta/3)$ is in the form $a \sin(b\theta)$, where $a = -2$ and $b = 2/3$.

Thus the amplitude of this function is $|a|$ (i.e., 2), and the period is

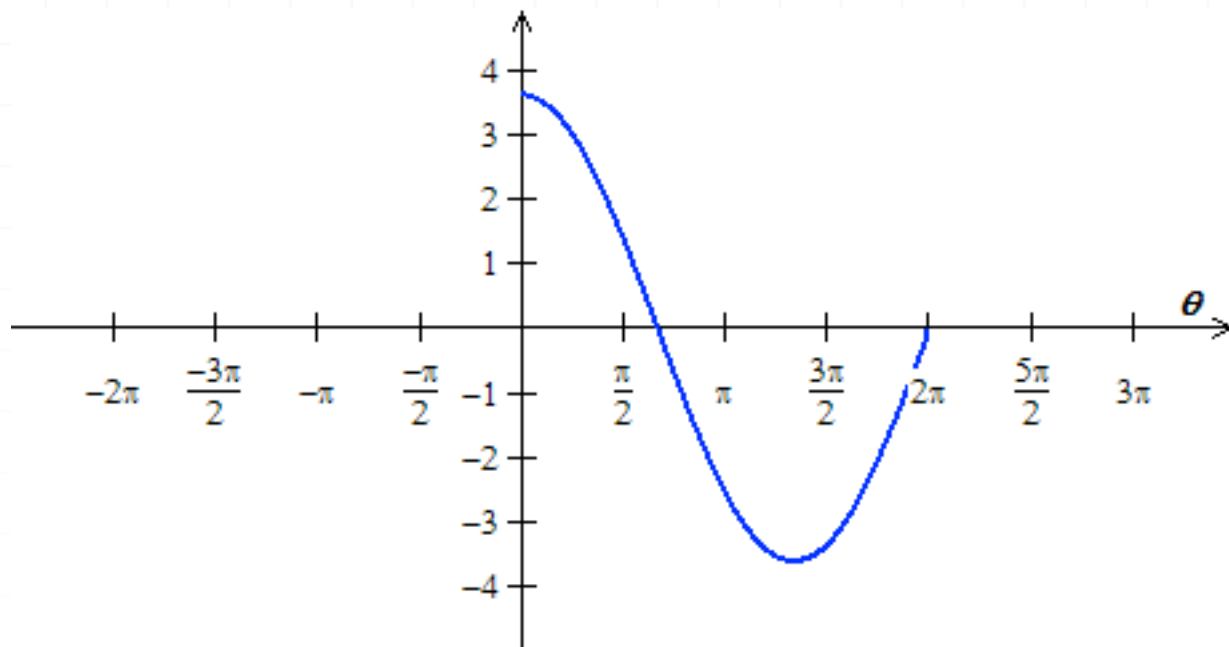
$$\frac{2\pi}{b} = \frac{2\pi}{\left(\frac{2}{3}\right)} = 2\pi \left(\frac{3}{2}\right) = 3\pi$$

This implies that the function $-2 \sin(2\theta/3)$ is equal to 0 at $\theta = 0$ and at the angle θ which is halfway through the interval $[0, 3\pi)$, that is, at $\theta = 3\pi/2$. The only curve that includes the point $(3\pi/2, 0)$ is the curve in answer choice D.



Topic: Sketching sine and cosine

Question: The following curve is the graph of which of the following functions?

**Answer choices:**

A $3.6 \cos\left(\frac{3\theta}{4}\right)$

B $3.6 \sin\left(\frac{6\theta}{5}\right)$

C $3.6 \cos\left(\frac{4\theta}{3}\right)$

D $-3.6 \sin\left(\frac{3\theta}{2}\right)$



Solution: A

Since the value of the function at $\theta = 0$ isn't equal to 0, the curve can't be the graph of either of the sine functions given in answer choices B and D. Thus all that remains is to show that the curve is the graph of the cosine function given in answer choice A (and not the graph of the cosine function given in answer choice C).

First, let's consider the function given in answer choice A: $3.6 \cos(3\theta/4)$. This function is in the form $a \cos(b\theta)$, where $a = 3.6$ and $b = 3/4$. Thus the period of this function is

$$\frac{2\pi}{b} = \frac{2\pi}{\left(\frac{3}{4}\right)} = 2\pi \left(\frac{4}{3}\right) = \frac{8\pi}{3}$$

This tells us that the value of $3.6 \cos(3\theta/4)$ is equal to 0 at the angles θ which are one-fourth and three-fourths of the way through the interval $[0, 8\pi/3)$. That is, $3.6 \cos(3\theta/4)$ is equal to 0 at

$$\theta = \left(\frac{1}{4}\right) \left(\frac{8\pi}{3}\right) = \left(\frac{1}{4}\right) \left(\frac{8\pi}{3}\right) = \frac{2\pi}{3}$$

and at

$$\theta = \left(\frac{3}{4}\right) \left(\frac{8\pi}{3}\right) = \left(\frac{3}{4}\right) \left(\frac{8\pi}{3}\right) = 2\pi$$

Now let's consider the function given in answer choice C: $3.6 \cos(4\theta/3)$. This function is in the form $a \cos(b\theta)$, where $a = 3.6$ and $b = 4/3$. Thus its period is



$$\frac{2\pi}{b} = \frac{2\pi}{\left(\frac{4}{3}\right)} = 2\pi \left(\frac{3}{4}\right) = \frac{3\pi}{2}$$

Thus the value of $3.6 \cos(4\theta/3)$ is equal to 0 at the angles θ which are one-fourth and three-fourths of the way through the interval $[0, 3\pi/2)$. That is, $3.6 \cos(4\theta/3)$ is equal to 0 at

$$\theta = \left(\frac{1}{4}\right) \left(\frac{2\pi}{b}\right) = \left(\frac{1}{4}\right) \left(\frac{3\pi}{2}\right) = \frac{3\pi}{8}$$

and

$$\theta = \left(\frac{3}{4}\right) \left(\frac{2\pi}{b}\right) = \left(\frac{3}{4}\right) \left(\frac{3\pi}{2}\right) = \frac{9\pi}{8}$$

Inspection of the curve shows that it includes neither the point $(3\pi/8, 0)$ nor the point $(9\pi/8, 0)$. However, the curve does include both the point $(2\pi/3, 0)$ and the point $(2\pi, 0)$. Therefore, the curve is the graph of the function given in answer choice A over the interval $[0, 2\pi)$.

